B.Sc. Part I (Hons.). ~~Ist~~ IInd Paper

INTEGRAL CALCULUS

(Reduction Formula) contd.

Q. Establish reduction formula for

$$\int \sin^m x \cos^n x \, dx.$$

Soln.

Let $$I_{m,n} = \int \sin^m x \cos^n x \, dx$$

$$\Rightarrow I_{m,n} = \int \cos x . \cos^{n-1} x \sin^m x \, dx$$

$$= \int \underbrace{(\cos^{n-1} x .)}_{u} \underbrace{(\cos x . \sin^m x \, dx)}_{v}$$

$$= \cos^{n-1} x \int \cos x \sin^m x \, dx - \int \left[\frac{d}{dx} (\cos^{n-1} x) \int \cos x \sin^m x \, dx \right] dx$$

$$\Rightarrow I_{m,n} = \cos^{n-1} x \int z^m dz + \int \left[(n-1) \cos^{n-2} x \sin x \int z^m dz \right] dx$$

where $z = \sin x \Rightarrow dz = \cos x \, dx$

$$\Rightarrow I_{m,n} = \cos^{n-1} x \frac{z^{m+1}}{m+1} + (n-1) \int \left[\cos^{n-2} x \sin x . \frac{z^{m+1}}{m+1} \right] dx$$

$$\Rightarrow I_{m,n} = \cos^{n-1} x \frac{\sin^{m+1} x}{m+1} + \frac{(n-1)}{m+1} \int \cos^{n-2} x . \sin x . \sin^{m+1} x \, dx$$

$$\Rightarrow I_{m,n} = \frac{\sin^{m+1} x}{m+1} \cos^{n-1} x + \frac{n-1}{m+1} \int \cos^{n-2} x \cdot \sin^m x \cdot \sin^2 x \, dx$$

$$= \frac{\sin^{m+1} x}{m+1} \cos^{n-1} x + \frac{n-1}{m+1} \int \cos^{n-2} x \cdot \sin^m x \, (1-\cos^2 x) dx$$

$$= \frac{\sin^{m+1} x}{m+1} \cos^{n-1} x + \frac{n-1}{m+1} \int \left[\cos^{n-2} x \sin^m x - \cos^n x \sin^m x \right] dx$$

$$\Rightarrow I_{m,n} = \frac{\sin^{m+1} x}{m+1} \cos^{n-1} x + \frac{n-1}{m+1} \int \cos^{n-2} x \sin^m x \, dx - \frac{n-1}{m+1} \int \sin^m x \cos^n x \, dx$$

$$\Rightarrow I_{m,n} = \frac{\sin^{m+1} x}{m+1} \cos^{n-1} x + \frac{n-1}{m+1} I_{m,n-2} - \frac{n-1}{m+1} I_{m,n}$$

$$\Rightarrow I_{m,n} \left[1 + \frac{n-1}{m+1} \right] = \frac{\sin^{m+1} x \cdot \cos^{n-1} x}{m+1} + \frac{n-1}{m+1} I_{m,n-2}$$

$$\Rightarrow I_{m,n} \times \left(\frac{m+n}{m+1} \right) = \frac{\sin^{m+1} x \cos^{n-1} x}{m+1} + \frac{n-1}{m+1} I_{m,n-2}$$

$$\Rightarrow I_{m,n} = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} I_{m,n-2}.$$